# Complex variables & Laplace Transformation

# Assignment 01

November 2, 2024

Total - 40 Marks

Due date: Wednesday, November - (Please submit hard copy)

(You need to answer all questions except bonus)

*bonus will be counted if you didn't get full mark

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1. (a) Express each of the following complex numbers in polar form,

$$(i) \quad 2+2\sqrt{3}, \quad (ii) \quad -5+5i$$

(b) Express each of the following complex numbers in Rectangular form,

$$(i) \quad 13\exp\left(\tan^{-1}\left(\frac{5}{12}\right)i\right), \quad (ii) \quad 4\left(\cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi\right) \quad \text{or} \quad r=4, \theta=-\frac{\pi}{3}$$

(c) Find the principal argument of the complex numbers from $(a)$.

(d) Find all the 10th roots of unity.

(e) Find the roots of $(-27i)^{1/6}$, locate them graphically.

(2+2+2+2+2 Marks)

2. (a) Describe graphically the region represented by each of the following,

$$(i) \quad Re(1/z) > 1 \quad (ii) \quad Re(z^2) > 1 \quad (iii) \quad Im(z^2) = 4 \quad (iv) \quad |\, z-3 \,| - |\, z+3 \,| = 4$$

(b) Solve the equation $z^2 + (2i-3)z + 5 - i = 0$

(12+3 Marks)

3. (a) Solve the following equations for $z \in \mathbb{C}$,

$$(i) \quad \cos(z) = -2i, \quad (ii) \quad \sin(z) = i$$

(5+5 Marks)

4. Prove De Moivre's theorem. And show that,

$$\sin(0\theta) + \sin(\theta) + \cdots + \sin(n\theta) = \frac{\sin(\frac{n}{2}\theta)\sin(\frac{n+1}{2}\theta)}{\sin(\frac{1}{2}\theta)}$$

(2.5+2.5 Marks)

**Bonus Question:**

1. Find the image of the unit square under the mapping $f(z) = (1+i)z + i$, (1 Marks)

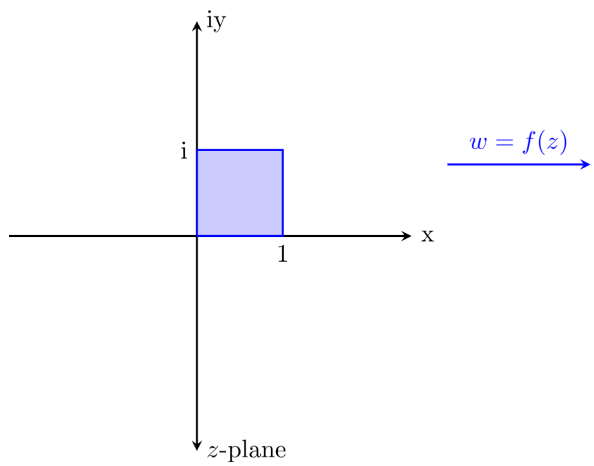


Figure 1: $f(z) = (1+i)z + i$

2. (a) Find the image of the given line under the given map,
    1. $Im(z) = 1; \quad f(z) = \cos(z)$
    2. $Re(z) = \frac{\pi}{6}; \quad f(z) = \sin(z)$

   (b) Find the image of the set $U$ under the function $f(z) = \sin(z)$.

$$U = \left\{ z \in \mathbb{C} : -\frac{\pi}{2} < Re(z) < \frac{\pi}{2} \right\}$$

(2+2 Marks)

## **Best of Luck!**