



MAT215: Complex Variables & Laplace Transformations

Quiz-04

January 7, 2026

Total - 20 Marks

(You need to answer **All questions**)

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**Name:**

**ID:**

**Section:**

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1. Evaluate the following integral using cauchy integral theorem:

$$\int_{|z|=3} \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$$

(10 Marks)

2. Compute

$$\int_C \frac{1}{z} dz$$

Where  $C$  is a square centred at the origin with the vertices  $\pm 1 \pm i$ .

3. Compute

$$\int_{|z|=3} \frac{z}{z^2 + 4} dz$$

(10 Marks)

**Bonus Question:**

1. Compute the integral

$$\int_{|z|=1} \frac{e^{z^2}}{z-2} dz$$

(1 Marks)

# 1 Formulas

**Cauchy's Integral Formula:**

Let  $f(z)$  be analytic inside and on a simple closed curve  $C$  and let  $z = z_0$  be any point inside  $C$ . Then,

$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z - z_0)^{n+1}} dz$$

**Residue Theorem:**

Let  $f(z)$  be single-valued and analytic inside and on a simple closed curve  $C$  except at the singularities  $z_1, z_2, z_3, \dots, z_k$  inside  $C$ . Then the residue theorem states that:

$$\oint f(z) dz = 2\pi i \sum_{i=1}^k \text{Re}(z = z_i)$$

where the Residue  $\text{Re}(z = z_i)$  can be calculated by,

$$\text{Re}(z = z_i) = \lim_{z \rightarrow z_i} \frac{1}{(m-1)!} \frac{d^{(m-1)}}{dz^{(m-1)}} \{(z - z_i)^m f(z)\}$$

where  $z = z_i$  is the Pole of order  $m$ .

**Best of Luck!**