MAT215: Complex Variables And Laplace Transformations

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LECTURE-02

Parametrization

The key to parametrization is to realize that the goal of this method is to describe the location of all points on a geometric object, such as a curve, a surface, or a region. This description must be one-to-one and onto: every point must be described once and only once.

$$\gamma := \{(x,y) : x^2 + y^2 = 1\}$$

If you want to geometrically analyze the curve γ (length/enclosed area, etc), we need a manageable way to produce the points (production scheme).

Motivation of Parametrization

Parametric curve: A curve in the 2-D plane can be described by,

$$\left.\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}\right\} \text{ where } a \leqslant t \leqslant b$$

These are called parametric equations for that curve, and t is the parameter.

What is the difference between general and parametric curves?

Example

Plot the parametric curves defined by,

$x = \cos t, y = -\sin t; 0 \leq t \leq \pi$

$x = t, y = 1; 0 \leq t \leq 4$

$z = 1 + it; 0 \leq t \leq 1$

Straight line from $(0,3)$ to $(2,3)$

Straight line from $(2,3)$ to $(2,4)$

Straight line from $(0,3)$ to $(2,4)$

Formula

Line segment: $\gamma(t) = a + t(b-a), 0 \leq t \leq 1$

Circle: $\gamma(t) = (a + r\cos(t), b + r\sin(t)), 0 \leq t \leq 2\pi$

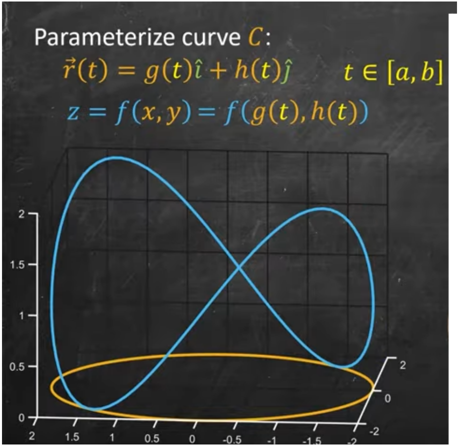
Explicit function: $\gamma(t) \overset{y=f(x)}{=} (t, f(t))$

Explicit function in polar: $\gamma(t) \overset{r=f(\theta)}{=} (f(t)\cos(t), f(t)\sin(t))$

Smooth Curves

Derivative exists at all points and is continuous

Motivation of line integral



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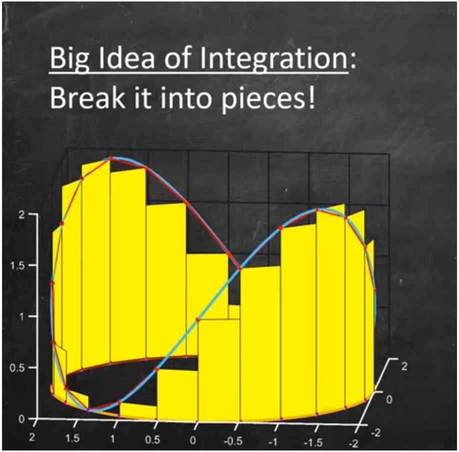
Big Idea of Integration:
Break it into pieces!
2
1.5
1
0.5
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
2
0
-2

Path in complex integration

Is there any meaning of

$$\int_{z_1}^{z_2} f(z)dz = ?$$

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Consider $C : z(t) = h(t) + ig(t); a \leq t \leq b$ Then line integral (or complex line integration) of f along C is defined as,

$$\int_C f(z)dz = \int_a^b f(\gamma(t))\gamma'(t)dt$$

Example

$C_1 : z(t) := \gamma(t) = t, 0 \leq t \leq 1$
$C_2 : z(t) = 1 + it, 0 \leq t \leq 1$
$f(z) = \bar{z}$ $C_3 : z(t) = \gamma(t) = t(1+i), 0 \leq t \leq 1$

Example

Evaluate $\int_{(0,1)}^{(2,5)}(3x+y)dx+(2y-\lambda$

along the parabola $y=x^2+1$

along the straight line from $(0,1)$ to $(2,5)$

along the straight lines from $(0,1)$ to $(0,5)$ and then from $(0,5)$ to $(2,5)$

Example

Evaluate $\oint_C(x+2y)dx+(y-2x)dy$ around the ellipse C defined by $x=4\cos\theta,\ y=3\sin\theta, 0\leq\theta\leq 2\pi$ if C is described in a counterclockwise direction.

Example

Evaluate $\int_C \left(x^2 - iy^2\right) \underline{dz}$

along the parabola $y = 2x^2$ from $(1,2)$ to $(2,8)$

along the straight line from $(1,2)$ to $(2,8)$

along the straight lines from $(1,2)$ to $(1,8)$ and then from $(1,8)$ to $(2,8)$

Example

Evaluate $\oint |z|^2 dz$ around the square with vertices at $(0,0),(1,0),(1,1)$ and $(0,1)$.

Example

Evaluate $\oint_C (\bar{z})^2 dz$ around the circle $|z-1| = 1$.