MAT215: Complex Variables And Laplace Transformations

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LECTURE-03

Polar Form

Problem

Find the Polar form of:

$-1+\sqrt{3}i$

$12i$

Find the expression:

$z_1 z_2 = ?$

$\frac{z_1}{z_2} = ?$

$z^{-1}?$

Hint: You need to pray to get the formula :3

Exponential Form

Find the expression:

$z_1 z_2 = ?$

$\frac{z_1}{z_2} = ?$

$z^{-1}?$

Hint: Use the polar form of each complex number and manipulate them.

Issues

Let for multiplication, $z_1 = i, z_2 = -1$ But

$$\text{Arg}(z_1) + \text{Arg}(z_2) = \frac{\pi}{2} + \pi = \frac{3\pi}{2} \neq -\frac{\pi}{2} = \text{Arg}(z_1 z_2)$$

To bring it back into the principal range, subtract.

Let for division, $z_1 = -i, ; z_2 = -1$. Then: $\text{Arg}(z_1) = -\frac{\pi}{2}, \text{Arg}(z_2) = \pi$.
Then:

$$-\frac{\pi}{2} - \pi = -\frac{3\pi}{2}$$

.

Now $-\frac{3\pi}{2} \notin (-\pi, \pi])$. But $\frac{3\pi}{2} + 2\pi = \frac{\pi}{2}$

Let for power, $z = -1$, so $\text{Arg}(z) = \pi$. Then $z^2 = (-1)^2 = 1$, and $\text{Arg}(z^2) = 0$.
But $n\,\text{Arg}(z) = 2\pi \notin (-\pi, \pi]$. We must subtract 2π to bring it back: $2\pi - 2\pi = 0$.

Problems

To find a large power or nth root, use exponential form.

Problem

Solve the equation: $e^{4z} = i$

Hint: *Start with known complex number*

Problem

Solve for x *and* y,

$$\left(\frac{3}{2} + \frac{\sqrt{3}}{2}i\right)^{2024} = 3^{1012}(x + iy)$$

Hint: *Start with known complex number*

Examples

Find $(1+i\sqrt{3})^{10}$, $(-16)^{\frac{1}{4}}$, all the cubic roots of unity.

Problem

Find all the values of z for which $z^5 = 32\sqrt{-1}$, and locate them in the complex plane.

Problems

Problem

Graph the lines,

$$|z| = 2$$

$$|z - 2i| = 3$$

$$\left|\frac{z-3}{z+3}\right| = 2$$

$$\text{Im}(z^2) = 4$$

$$\text{Re}(z^2) = 4$$

$$\text{Re}\left(\frac{1}{z^2}\right) = 1$$

$$\arg(z) = \frac{\pi}{3}$$

Problems

Problem

Graph the regions,

$$|z| > 2$$

$$|z - 2i| \leq 3$$

$$\left|\frac{z-3}{z+3}\right| < 2$$

$$\text{Im}(z^2) > 4$$

$$\text{Re}(z^2) < 4$$

$$\frac{\pi}{4} \leq \arg(z) \leq \frac{2\pi}{3}$$

Issues

$$|z_1 z_2| = |z_1|\,|z_2| \text{ but } |z_1 + z_2| \leq |z_1| + |z_2|$$

$$\sqrt{z_1 z_2} \neq \sqrt{z_1}\,\sqrt{z_2}$$

$$(z^a)^b \neq z^{ab}$$

$$\log_a b^r \neq r \log_a b$$

Consider the following examples: $z_1 = 1 + i, z_2 = -1 + i$,
$\sqrt{(-1)(-1)} \neq \sqrt{(-1)}\sqrt{(-1)}$.

Complex Power