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Department of Mathematics and Natural Sciences
MAT216 : Linear Algebra and Fourier Analysis
Assignment 2
Date: [redacted]

Deadline: [redacted] [redacted] Total Marks: 80

Name:

Section:

ID:

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1. Determine which sets are vector spaces under the given operations. For those that are not vector spaces, list at least one axiom that fail to hold. $[2.5 \times 5 = 12.5]$

   a. The set of all triples of real numbers (x,y,z) with the operations $(x,y,z)+(x',y',z') = (x+x',y+y',z+z')$ and $k(x,y,z) = (0,0,0)$.
   b. The set of all pairs of real numbers of the form (x,y), where $x \geq 0$; with the standard operations on $\mathbb{R}^2$.
   c. The set of all pairs of real numbers (x,y) with the operations $(x,y)+(x',y') = (x+x'+\ 1, y+y'+1)$ and $k(x,y) = (kx,ky)$.
   d. The set of all pairs of real numbers of the form $(1,x)$ with the operations $(1,y)+(1,y') = (1,y+y')$ and $k(1,y) = (1,ky)$.
   e. The set of all positive real numbers with the operations $x+y = xy$ and $kx = x^k$.

2. Determine which of the following are subspace of the vector space V. [1.5 × 5 = 7.5]

   a. All vectors of the form $(a,0,0)$, where $V = \mathbb{R}^3$
   b. All vectors of the form (a,b,c) with $c = a-b$, where $V = \mathbb{R}^3$
   c. All vectors of the form (a,b,c) with $c = a+b+3$, where $V = \mathbb{R}^3$
   d. All matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $a+b+c+d = 0$, where $V = M_{2\times 2}$
   e. All matrices $\begin{bmatrix} a & a \\ -a & -a \end{bmatrix}$, where $V = M_{2\times 2}$

3. Find two different bases of the subspace [5]

$$V = \text{Span}\left\{ \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} \right\}.$$

4. In $V = \mathbb{R}^4$, consider the vectors [5]

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ -2 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -1 \\ 4 \\ 2 \\ -3 \end{bmatrix}.$$

Let, $\mathrm{W} = \mathrm{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Is $\mathscr{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ a basis for W ?

5. (i) Find a basis for the set of 3 × 3 skew-symmetric matrices. (ii) Find a basis for the set of 3 × 3 symmetric matrices. [2.5 × 2 = 5]

6. Find a basis for Row(A), Col(A) and Null(A) where [5]

$$\mathbf{A} = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$$

7. Find the rank and nullity of the matrix [5]

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 2 & 3 & -6 \\ 0 & -1 & -3 & 1 & 1 \\ -2 & 4 & -3 & -6 & 11 \end{bmatrix}$$

8. Let [5]

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix}$$

and let $\mathscr{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$. Show that $\mathscr{B}$ is linearly independent and therefore a basis for $\mathrm{W} = \mathrm{span}\{\mathbf{v}_1, \mathbf{v}_2\}$. Determine if $\mathbf{x}$ is in W, and if so, find the coordinate vector of $\mathbf{x}$ relative to $\mathscr{B}$.

9. Consider the transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$, where, [10]

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x-y \\ y \end{bmatrix}$$

(a) Show that T is linear.
(b) What is the standard matrix corresponding to T ?
(c) Calculate the $\dim(\mathrm{Img}(T))$ and hence $\mathrm{rank}(T)$
(d) Calculate $\dim(\mathrm{Ker}(T))$ and hence $\mathrm{nullity}(T)$.

10. Find the eigenvalues and corresponding eigenvectors of the matrix, [5]

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

11. Determine whether A is diagonalizable or not. If diagonalizable, find P such that $P^{-1}AP$ is diagonal. Also calculate A^{15}. [5]

$$A = \begin{bmatrix} 7 & 5 \\ -1 & 1 \end{bmatrix}$$

12. Geometrically explain why any two two-dimensional linearly independent vectors is a basis for $\mathbb{R}^2$? Does this apply to any higher dimension? [5]

13. Answer in short:

(a) How to determine the linear independence or dependence of two vectors in any vector space (where the addition and scalar multiplication are defined in the usual manner) directly without solving a linear system? [2]

(b) Suppose there are three vectors none of which is a linear combination of the other two. What's the conclusion? Does this idea apply to any number of vectors? [2]

(c) Does the conclusion of the portion (a) remain the same when the operations are not usual? [1]