1 Eigenvalues, Eigenvectors and Diagonalization

1. Find the eigenvalue and eigenvector pair for the following matrix,

$$A = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix}$$

And diagonalize it.

2. Find the eigenvalue and eigenvector pair for the following matrix,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

And diagonalize it.

3. Find the eigenvalue and eigenvector pair for the following matrix,

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And diagonalize it.

Answer: In this example we will see the eigenvalues are complex. In fact, we can find the complex eigenvectors for this case. But don't worry we won't encounter such a problem in the final :3

4. Find A^{999} for the following matrix,

$$A = \begin{bmatrix} 7 & -3 \\ -2 & 8 \end{bmatrix}$$

2 Fourier (Sine/Cosine) Series

- 5. Determine the period of the function $y = 7 \sin^{2024} 5x + 21$.
- 6. Draw sketches of the following graphs:
 - $f(x) = 3|\sin(x)|, 0 \le x \le 2\pi$

- $\begin{cases} 2\sin x &, 0 \le x \le \pi \\ 0 &, \pi \le x \le 2\pi \end{cases}$ • $\begin{cases} 1 &, 0 \le x \le \pi \\ 0 &, \pi \le x \le 2\pi \end{cases}$
- 7. Find the Fourier series expansion of the function f(x),

$$f(x) = \begin{cases} x & , 0 < x < \pi \\ -x & , -\pi < x < 0 \end{cases}$$

8. Find the Fourier series expansion of the function f(x),

$$f(x) = \begin{cases} 2 & , 0 < x < \pi \\ -2 & , -\pi < x < 0 \end{cases}$$

9. Find the Fourier series expansion of the function f(x),

$$f(x) = 3|\sin x|, 0 \le x < 2\pi$$

10. Find the Fourier series expansion of the function f(x),

$$f(x) = \begin{cases} 2 & , 0 < x < \pi \\ -2 & , -\pi < x < 0 \end{cases}$$

11. Find the Fourier Sine series expansion of the function f(x),

$$f(x) = \cos x, 0 \le x \le \pi$$

12. (skip this question, it contains some issues) Find the Fourier series expansion (complex form) of the function f(x),

$$f(x) = \begin{cases} 1 & , |x| \le 1 \\ 0 & , \text{ otherwise} \end{cases}$$

- 13. Find the odd extension of $f(x) = \cos(x)$ on $[0, \pi]$, draw it and find the Fourier Sine Series of it.
- 14. Find the even extension of f(x) = 1 x on $[0, \pi]$, draw it and find the Fourier Sine Series of it.

3 Fourier (Sine/Cosine) Transform

15. Given that,

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Use Fourier transform to find,

$$\int_{-\infty}^{\infty} \frac{\sin(\omega a)\cos(\omega x)}{\omega} d\omega$$

Use your result to find,

$$\int_{-\infty}^{\infty} \frac{\sin(x)}{x} dx = \frac{\pi}{2}$$

16. Given that,

$$f(x) = \begin{cases} 1 - x, & |x| < 1\\ 0, & |x| > 1 \end{cases}$$

Use Fourier transform to find,

$$\int_0^\infty \left(\frac{x\cos(x) - \sin(x)}{x^3}\right) \cos\frac{x}{2} dx$$

17. Given that,

$$f(x) = \begin{cases} 1, & 0 < x < 1\\ 0, & x \ge 1 \end{cases}$$

Find Fourier Sine and Cosine transform of f(x).

Best of Luck!