# Linear Algebra & Fourier Analysis

## Assignment 01

June 18, 2024

Total - 50 Marks, Due date: Thursday, June 27, Please submit hard copy
(You need to answer all questions except bonus)

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1. Solve the following system where $a$, $b$ and $c$ are constants:

$$\begin{aligned} x_1 + x_2 + x_3 &= a \\ 2x_1 + 2x_3 &= b \\ 3x_2 + 3x_3 &= c \end{aligned}$$

   Hint: Apply Gaussian Elimination (5 Marks)

2. What condition, if any, must $a$, $b$ and $c$ satisfy for the linear system to be consistent?

$$\begin{aligned} x_1 + 3x_2 + x_3 &= \alpha \\ -x_1 - 2x_2 + x_3 &= \beta \\ 3x_1 + 7x_2 - x_3 &= c \end{aligned}$$

   Hint: Apply Gaussian Elimination (5 Marks)

3. Find the reduced row echelon form of the augmented matrix for the linear system:

$$\begin{aligned} 6x + y + 4t &= -3 \\ -9x + 2y + 3z - 8t &= 1 \\ 7x - 4z + 5t &= 2 \end{aligned}$$

   Can you identify the pivot columns? Write the pivot columns. (5 Marks)

4. Take your student **ID** as a number. Let **ID4** represent the 4th digit of your **ID**. For example, if 87654321 is your **ID** then **ID2** is 7.

   (a) Now write the corresponding system of equation of the following augmented matrix:

$$\left(\begin{array}{cccc|c} \mathbf{ID8} & 0 & 0 & \mathbf{ID8} & \mathbf{ID8} \\ 0 & \mathbf{ID7} & \mathbf{ID6} & 0 & \mathbf{ID7} \\ \mathbf{ID3} & \mathbf{ID7} & \mathbf{ID6} & \mathbf{ID3} & \mathbf{ID3}+\mathbf{ID7} \end{array}\right)$$

   (b) Find the parametric solution as well as the general solution (if possible) of that system.

   (c) Find two random solution $\mathbf{u}$ and $\mathbf{v}$ and show that if $\mathbf{u}+\mathbf{v}$ is also a solution of the system or not?

(2+7+1 Marks)

5. Find the inverse of the following matrix using Gaussian Elimination:

$$\begin{pmatrix} \mathbf{1} & 0 & 0 & \mathbf{ID5} \\ 0 & \mathbf{2} & \mathbf{ID6} & 0 \\ 0 & \mathbf{ID7} & \mathbf{3} & 0 \\ \mathbf{ID8} & 0 & 0 & \mathbf{4} \end{pmatrix}$$

(15 Marks)

6. (a) Find the values of $k$ when the following systems will be consistent:

$$\left(\begin{array}{cc|c} 3 & -4 & k \\ -6 & 8 & 5 \end{array}\right), \left(\begin{array}{cc|c} k & 1 & -2 \\ 4 & -1 & 2 \end{array}\right)$$

   (b) Find the value of $\lambda$ and $\mu$ such that the following system of equations have $(i)$ No solution, $(ii)$ Many solution, and $(iii)$ Unique solution (if possible):

$$\begin{cases} x+y+z &= 6 \\ x+2y+3z &= 10 \\ x+2y+\lambda z &= \mu \end{cases}, \qquad \begin{cases} x+y+z &= 6 \\ y+2z &= 4 \\ (\lambda-3)z &= \mu-10 \end{cases}$$

(10 Marks)

**Bonus Question:**

1. (a) Let $t$ be an arbitrary real number and let,

$$
\begin{aligned}
s_1 &= -\frac{3}{2} - 2t \\
s_2 &= \frac{3}{2} + t \\
s_3 &= t
\end{aligned}
$$

Show that for any choice of the parameter $t$, the list $(s_1, s_2, s_3)$ is a solution to the linear system

$$
\begin{aligned}
x_1 + x_2 + x_3 &= 0 \\
x_1 + 3x_2 - x_3 &= 0
\end{aligned}
$$

(b) Suppose we have given the null space (remember I introduced it as nullifying vectors which appear inside our general solution) of a matrix which was in the reduced row echelon form:

$$
N = \alpha \begin{pmatrix} 7 \\ 0 \\ 0 \\ 8 \\ 0 \\ 0 \\ 3 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 0 \\ 0 \\ 4 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 2 \\ 0 \\ 0 \\ 3 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

Now, can you predict that linear system?
Hint: Can you identify the pivot columns? Look carefully at the nullifying vectors and what information they contain (like how they interact with the pivots).

(2+3 Marks)

**Best of Luck!**