BRAC UNIVERSITY
Inspiring Excellence

Linear Algebra & Fourier Analysis
Quiz-01

June 27, 2024

Total - 40 Marks

(You need to answer all questions)

Name:

ID:

Section:

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1. (a) Solve the linear system (using Gaussian Elimination or Gauss-Jordan method):

$$\begin{cases} x + 3y + 2z &= 2 \\ 2x + 7y + 7z &= -1 \\ 2x + 5y + 2z &= 7 \end{cases}$$

Hint: You can reach row echelon form within 3 steps or reduced row echelon form within 6 steps.

(b) Suppose you have the following augmented matrix and the reduced row echelon form of that matrix respectively. Can you determine under what condition for a, b and c the system is consistent and inconsistent?

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -3 & a \\ 1 & 1 & 3 & 1 & b \\ 1 & 0 & 2 & -1 & c \end{array}\right) \sim \cdots \sim \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & c \\ 0 & 1 & 1 & 2 & -a+c \\ 0 & 0 & 0 & 0 & a+b-2c \end{array}\right)$$

Hint: Come on. The answer is on the paper. Just write it down.

(c) Now, if we consider the columns as the vectors $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ and $v_4 = \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$ in $\mathbb{R}^3$. Can $w = \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix}$ be written as a linear combination of v_1, v_2, v_3 and v_4?

Hint: Linear combination mean $w = \alpha v_1 + \beta v_2 + \gamma v_3 + \mu v_4$. Insert those vectors in this equation. Did you see something similar to (b)? What can you say if we consider $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix}$.

(10+2+3 Marks)

2. (a) Using Row Operations to find A^{-1}:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$$

(15 Marks)

3. Write **True** if the statement is correct otherwise **False**. A short explanation is needed to justify your answer.

   (a) If a linear system has more unknowns than equations, then it has infinitely many solutions.

   (b) If the number of equations in a linear system exceeds the number of unknowns, then the system must be inconsistent.

   (c) The linear system,

$$\begin{cases} x - y & = 3 \\ 2x - 2y & = k \end{cases}$$

   can't have a unique solution, regardless of the value of k.

   (d) The linear system with corresponding augmented matrix

$$\left(\begin{array}{cc|c} 2 & -1 & 4 \\ 0 & 0 & -1 \end{array} \right)$$

   is consistent.

   (e) If A is an $n \times n$ matrix that is not invertible, then the linear system $A\mathbf{x} = 0$ has infinitely many solution.

(10 Marks)

Best of Luck!